KSME Journal, Vol. 2, No. 1, pp. 3~8, 1988.

ESTIMATION OF THE LEAKAGE FOR COMPRESSIBLE GASES IN HIGH-SPEED SHAFT SEALS

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(*Received July 25, 1987*)

A comprehensive analytical work to estimate the leakage rate is presented for compressible fluid flow across shaft seals. The sealing gap for this study includes geometric terms such as eccentricity, misaligned shaft, and sinusoidal waviness of the mating surfaces. A temperature distribution across the sealing gap is developed using a temperature dependent viscosity. A pressure distribution in polynomial form is solved based on the simplified nonlinear Reynolds equation using the approximate power series expansion. It was found that the seal performance is largely influenced by the eccentricity and width of the seal at high speeds (greater than about 150m/s).

Key Words: Shaft Seal, Sealing Gap, Power Series Technique, Leakage Flow Rate, Eccentricity

NOMENCLATURE

a : Velocity of sound
c_l : Wave velocity of the lower body
c_u : Wave velocity of the upper body
e : Eccentricity $(=\overline{O_1O_2})$
e_0 : Shaft eccentricity at $z=0(=\overline{O_1N})$
k : Specific heat ratio
K : Thermal conductivity
L : Half of the seal width
$\dot{m}$: Mass flow rate
$\dot{M}$: Dimensionless mass flow rate
N : Center at the midplane, i.e., $z=0$
O_1 : Center of the seal
O_2 : Center of the shaft at any plane
p : Pressure
$\bar{R}$: r/r_l
R_g : Gas constant
t : Time
T : Temperature
U : $r_2\omega$, velocity of the shaft
v_θ : Tangential velocity component of fluid
v_z : Axial velocity component of fluid
z : Axial coordinate
γ : Angle of tilt
ε : $\gamma L/\bar{h}$, i.e. $\bar{h}$ tilt parameter of the shaft seal
ε_0 : $e_0/\bar{h}$, eccentricity ratio at $z=0$
η : Viscosity of fluid
χ : $2\pi/\lambda$, wave number
λ : Wavelength
Λ_l : T_l/T_r
μ : Coefficient of friction
ξ_l : $|\bar{h}_g|/\bar{h}$
ξ_u : $|\bar{h}_u|/\bar{h}$
ρ : Density
Ω : Angular speed
Ω_l : $\chi_l c_l$
Ω_u : $\chi_u(c_u+U)$
ω : Angular speed of the moving surface

Subscripts
1 : Seal
2 : Shaft
r : Reference conditions
l : Lower
u : Upper

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1. INTRODUCTION

Eccentricity, misalignment, and wavy surface on the contacting surfaces of shaft seals may be the major components that largely affect the seal performance. In actual practice, it is very likely that these components are combined. However, the effects of the components are treated separately because of its complexity in the literature.

The geometry of misaligned shaft has been studied by Sassenfeld and Walther(1954). Several investigators have studied the effects of a misaligned shaft on the seal performance(Adams and Colsher, 1969; Chen and Jachson, 1985; Dhagat, Sinhasan and Singh, 1981; Fleming, 1979). The wavy model of face seals on the contacting surfaces was analyzed by Bryant and Kim(1987). Zuk(1973) addressed that pressure profiles are independent of fluid properties for surfaces with small linear tilts.

The sealing gap must be small enough so that the leakage flow rate is minimal. But it must be large enough so that the power loss due to viscous friction is very small. To satisfy the contradicting situations, it is necessary to consider all the possible elements of the sealing gap for an accurate estimation of pressure distribution and mass leakage rate through the sealing gap.

In this paper, a mathematical analysis of compressible fluid flow across shaft seals has been presented for various combinations of the geometric components. The nonlinear Reynolds equation was analytically solved using a power series technique.

2. ANALYSIS OF SEALING GAP

The seal performance is coupled to the determination of the accurate sealing. The overall sealing gap, h around the sealing circumference may then be described by a function of the form

$$h(g,\ z,\ t)=\bar{h}+\tilde{h}_t+\tilde{h} \tag{1}$$

where $\bar{h}$ is the radial sealing gap for the concentric shaft seal defined as $\bar{h}=r_1-r_2$, $\tilde{h}_t$ represents the sealing gap for the eccentricity and misaligned geometry of the shaft seal, and $\tilde{h}$ denotes the sealing gap variation due to wavy surfaces between the rotating shaft and the seal ring.

2.1 Sealing Gap due to Eccentricity and Misalignment

In most practical seals, the shaft is slightly bent and deviated from the center line of the seal ring due to a load being placed between two supporting parts such as a bearing. The sealing gap variation is associated with the distance between two eccentric cylindrical surfaces with an angular misalignment of the shaft as shown in Fig. 1. We shall consider the axis of the shaft to be straight through tilted. The sealing gap change, h_t of the eccentric and misaligned shaft, at any point of the circumferential direction is determined by

$$(r_2+h_t)^2=e^2+r_1^2+2er_1\ \cos(\theta+\phi) \tag{2}$$

where θ denotes the angular coordinate measured from the line of centers in the midplane. ϕ is an angle measured from the line of centers in the midplane to that at any other plane. e is the eccentricity measured along the line of centers between the shaft and the seal at any plane except the central plane of the shaft. The eccentricity is assumed to be a small fraction of r_1. Eq. (2) can then be simplified using the eccentricity ratio, $\tilde{\varepsilon}=e/\bar{h}$

$$h_t=\bar{h}[1+\tilde{\varepsilon}\ \cos(\theta+\phi)] \tag{3}$$

From the triangle NO_1O_2 of Fig. 1(b)

$$e^2=e_0^2+\overline{NO}_2{}^2+2e_0\overline{NO}_2\cos\ \Psi \tag{4}$$

where Ψ is the attitude angle between load line and line of centers in the midplane. From the Figure 1(b), ϕ using dimensionless axial coordinate, Z defined as $(z+L)/2L$, is given by

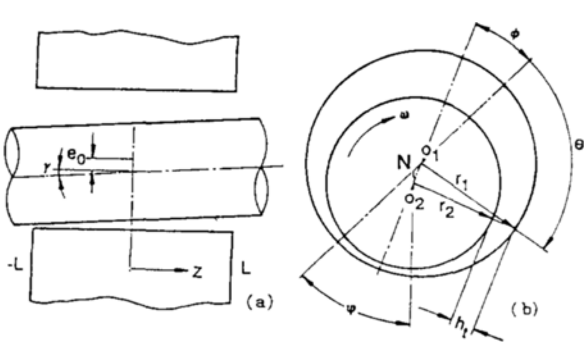

Fig. 1 Geometry of a misaligned shaft seal

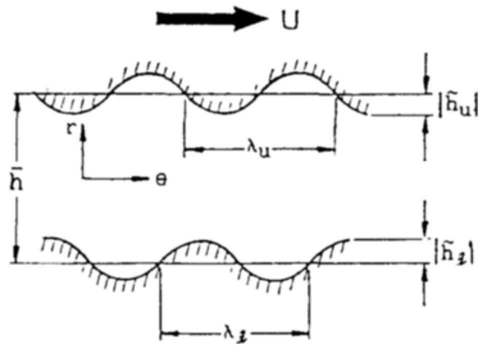

Fig. 2 Surface waviness of the shaft seal

$$\phi=\sin^{-1}\left[\frac{\varepsilon(2Z-1)}{\tilde{\varepsilon}}\ \sin\ \Psi\right] \tag{5}$$

where $\overline{NO}_2=-\gamma L(2Z-1)$

Equation (4) in terms of Z, eccentricity ratio ε_0, and tilt parameter ε, can be rewritten as

$$\tilde{\varepsilon}^2=\varepsilon_0^2+[\varepsilon(2Z-1)]^2-2\varepsilon_0\ \varepsilon(2Z-1)\ \cos\ \Psi \tag{6}$$

Substituting Eq. (5) into Eq. (3) for the small tilt angle gives

$$h_t=\bar{h}[1+\varepsilon_0\cos\theta-\varepsilon(2Z-1)\ \cos(\theta-\Psi)] \tag{7}$$

The above expression does not include the surface waviness. The first term on the right hand side is the normal sealing gap for the concentric case. The second term represents the eccentric effects of the parallel shaft seals. The last one denotes the sealing gap variation due to the angle of tilt.

2.2 Sealing Gap Variations Caused by Sinusoidal Surface Waviness

The sinusoidal waviness on the shaft and seal surfaces is considered. For the purpose of the analysis, the shaft and seal ring can be opened up into a blade of seal width, $2L$ in Fig. 2. The sealing gap change, $\tilde{h}$ due to the sinusoidally wavy surfaces is then given by

$$\tilde{h}=-|\tilde{h}_l|\sin(n_l\theta+\phi-\Omega_l t)-|\tilde{h}_u|\sin(n_u\theta+\phi-\Omega_u t) \tag{8}$$

where $|\tilde{h}_l|$ and $|\tilde{h}_u|$ are the amplitudes of the surface waviness at the shaft and seal, respectively. n is the number of waves around the shaft and seals.

3. ANALYSIS OF THE SHAFT SEAL ACROSS THE SEALING GAP

3.1 Temperature Distribution

The heat conduction along the normal to the sealing gap is assumed to be the major mechanism of heat transfer from (or to) the seal surfaces. It is assumed that the specific heat and the Prandtl number are constants for gases. The radial velocity is negligibly small for the very thin film between the two plates. The temperature is assumed to be axially symmetric. The energy equation in cylindrical coordinates for steady state flow of a compressible fluid may be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)-\rho c_p v_z\frac{\partial T}{\partial z}=$$
$$-\frac{1}{r}\frac{\partial}{\partial r}\left\{\eta\left[v_\theta\frac{\partial}{\partial r}\left(rv_\theta\right)+rv_z\frac{\partial v_z}{\partial r}\right]\right\}$$
$$+\frac{4\eta v_\theta}{r}\frac{\partial v_\theta}{\partial r} \tag{9}$$

The energy equation can be rewritten using the Reynolds number ($Re=hU/\nu_r$), Prandtl number ($Pr=\rho c_p/K$), and Eckert number [$Ec=(k-1)Ma^2 T_u/(T_l-T_u)$], and the following dimensionless parameters

$$V_\theta=v_\theta/U \qquad V_z=v_z/U \tag{10a}$$
$$R=(r-r_u)/(r_l-r_u) \qquad Z=(z+L)/2L \tag{10b}$$
$$\bar{T}=(T-T_u)/(T_l-T_u) \qquad \bar{\rho}=\rho/\rho_r \tag{10c}$$
$$\bar{\eta}=\eta/\eta_r \tag{10d}$$

Therefore the energy equation in dimensionless form becomes

$$\frac{1}{\tilde{R}}\frac{\partial}{\partial R}\left(R\bar{\eta}\frac{\partial\bar{T}}{\partial R}\right)+\left(\frac{h}{2L}\right)^2\frac{\partial}{\partial Z}\left(\bar{\eta}\frac{\partial\bar{T}}{\partial Z}\right)$$
$$-\frac{Uh^2}{2v_rL}Pr\left(\bar{\rho}V_z\frac{\partial\bar{T}}{\partial Z}\right)=$$
$$PrEc\left[-\frac{\partial}{\partial R}\left\{\bar{\eta}\left[(1-\tilde{R}_u)V_\theta+\frac{\tilde{R}_u}{2}\frac{\partial}{\partial R}\left(V_\theta^2+V_z^2\right)\right]\right\}\right.$$
$$\left.+4(1-\tilde{R}_u)\left(\frac{\bar{\eta}V_\theta}{\tilde{R}}\frac{\partial V_\theta}{\partial R}\right)\right] \tag{11}$$

where $Ma=U/a$ is the Mach number. It is assumed that the sealing gap, h is much less than the width of the seal, i.e., $h/2L\ll1$. As the radius ratio, $\tilde{R}_u$ approaches unity, terms of the order of $(1-\tilde{R}_u)$ become very small. If the temperature difference between two seal surfaces is large the pressure and dissipation terms may be negligible (Constantinescu, Smith and Pascovici, 1980). The viscosity of a compressible fluid may be expressed as a function of the temperature (Constantinescu, 1969).

$$\eta=\eta_r(T/T_r)^m \tag{12}$$

where m is determined by the property of a fluid.

A simplified energy equation for the negligible seal curvature is then obtained

$$\frac{\partial}{\partial r}\left(\eta\frac{\partial T}{\partial r}\right)=0 \tag{13}$$

Based on the data given in a later section, we introduce an error of approximately 4~6% by neglecting the dissipation term. The boundary conditions for Eq. (13) are given by

$$T=T_u, \quad \text{at } r=r_u=r_2+\tilde{h}_u \tag{14a}$$
$$T=T_l, \quad \text{at } r=r_l=r_2+h_l-\tilde{h}_l \tag{14b}$$

Integration of Eq. (13) using the expressions for viscosity (12) and the boundary conditions (14) yields the temperature distribution across the sealing gap

$$\beta=\left[\left(1-\beta_A^{m+1}\right)\frac{r}{h}+\frac{\tilde{R}_u-\beta_A^{m+1}}{\tilde{R}_u-1}\right]^{\frac{1}{m+1}} \tag{15}$$

with $\beta=T/T_l$, $\beta_A=T_u/T_l$, and $\tilde{R}_u=r_u/r_l$.

3.2 Velocity Distribution

In a similar manner some assumptions for the temperature distribution can be made for the velocity profiles. The body force is negligible. It is clear that the order of magnitude of the inertia forces of the equation of motion in dimensionless form depends on the Reynolds number. The condition to neglect the inertia forces for the shaft seal is that the Reynolds number must be less than the ratio of the width of the shaft seal to the sealing gap, i.e., $Re\ll 2L/h$. The effect of the surface curvature can be neglected owing to the small sealing gap. By examing the various terms of the Navier-Stokes equations with their relative order of magnitude it may then be greatly simplified for the steady conditions and a compressible laminar fluid flow.

$$\frac{\partial p}{\partial r}=0 \tag{16a}$$
$$\frac{1}{r}\frac{\partial p}{\partial\theta}=\frac{\partial}{\partial r}\left(\eta\frac{\partial v_\theta}{\partial r}\right) \tag{16b}$$
$$\frac{\partial p}{\partial z}=\frac{\partial}{\partial r}\left(\eta\frac{\partial v_z}{\partial r}\right) \tag{16c}$$

For the problem (16c) under consideration, the boundary conditions are

$$v_z=0, \quad \text{at } r=r_u \tag{17a}$$
$$v_z=0, \quad \text{at } r=r_l \tag{17b}$$

By using Eq. (15) and applying the boundary conditions (17), the velocity distribution in the axial direction can be written as

$$v_z=\frac{h^2}{\eta_r}\left(\frac{\partial p}{\partial z}\right)f(\tilde{R}) \tag{18}$$

where

$$f(\tilde{R})=\frac{G_1}{1-\tilde{R}_u}\left(\beta\tilde{R}-1-(\beta_A\tilde{R}_u-1)f_1(\tilde{R})\right) \tag{19a}$$
$$f_1(\tilde{R})=\frac{\beta-1}{\beta_A-1} \tag{19b}$$
$$G_1=\frac{m+1}{\Lambda_l^m\left(1-\beta_A^{m+1}\right)} \tag{19c}$$

3.3 Pressure Distribution

With the aid of the ideal gas equation, the temperature and velocity distributions of the shaft seal, the generalized form of Reynolds equation for variable viscosity and density is obtained by integrating the continuity equation across the sealing gap. The allowance for such variation in viscosity and density destroys the linearity of the equation and increases complexity.

The application of the narrow bearing approximation leads to the simplification of the Reynolds equation. As the width of the shaft seal becomes small compared with the outside radius of the shaft seal, the circumferential pressure gradient can be neglected in comparsion to the axial one in the dimensionless form of the modified Reynolds equation. The dimensionless equations are

$$P=\frac{r_l\bar{h}^2}{4\eta_rUL^2}p \qquad \bar{H}=\frac{h}{\bar{h}} \qquad R_u=\frac{r_u}{\bar{h}} \tag{20}$$

The normalized form of the modified Reynolds equation can then be written as

$$\frac{\partial}{\partial z}\left(\Gamma_1 P\hat{H}^3\frac{\partial P}{\partial z}\right)=-\left[\frac{\partial}{\partial\theta}\left(\Gamma_2 P\hat{H}\right)+\frac{P}{\Lambda_u}\frac{\partial R_u}{\partial\theta}\right] \tag{21}$$

where

$$\Gamma_1=\frac{G_1}{\Lambda_l(1-\tilde{R}_u)}\left\{\frac{1+\tilde{R}_u+\tilde{R}_u^2}{3}-\frac{1}{\beta_A-1}\left[\frac{(m+1)\beta_A(1-\tilde{R}_u)}{m(1-\beta_A^{m+1})}\left(1-\tilde{R}_u\beta_A^m-\frac{(m+1)(1-\tilde{R}_u)(1-\beta_A^{2m+1})}{(2m+1)(1-\beta_A^{m+1})}\right)+\frac{(1+R_u)(\tilde{R}_u\beta_A-1)}{2}\right]\right\} \tag{22a}$$

$$\Gamma_2=\frac{1}{\Lambda_l(\beta_A-1)}\left[1-\frac{(m+1)(1-\beta_A^m)}{m(1-\beta_A^{m+1})}\right] \tag{22b}$$

A power series technique in obtaining the solution of nonlinear equation (21) is useful for the complicated Reynolds equation(6). We assume that the pressure and sealing gap is given in the form of the series

$$P(Z,\theta,t)=\sum_{i=0}^{\infty}P_iZ^i \tag{23}$$

futhermore

$$\hat{H}(Z,\theta,t)=\sum_{i=0}^{N}\hat{H}_iZ^i \tag{24}$$

where $0\le Z\le 1$. The coefficients of Eq. (23) will be determined so that the governing equation (21) and its boundary conditions (28) will be satisfied.

Substituting Eq. (23) and (24) into Eq. (21) and collecting terms of like power of Z gives a system of equations

$$\sum_{k=0}^{j}\left\{\sum_{i=0}^{k}\sum_{m=0}^{j-k}\sum_{n=0}^{m}\left[3(k-i+1)(j-k-m+1)\cdot\hat{H}_{j-k-m+1}P_iP_{k-i+1}+(i+1)(k-i+1)\cdot\hat{H}_{j-k-m}P_{i+1}P_{k-i+1}+(k-i+1)(k-i+2)\cdot\hat{H}_{j-k-m}P_iP_{k-i+2}\right]\hat{H}_nH_{m-n}-A_kP_{j-k}\right\}=0 \tag{25}$$

In order that the equation (25) in terms of Z be valid over an interval, the coefficients of all powers of Z need to vanish independently. This generates a recursive formula for P_j given by

$$P_{j+2}=-S_j/(j+1)(j+2)\hat{H}_0^3P_0 \tag{26}$$

where

$$S_j=\sum_{k=0}^{j}\left\{\sum_{i=0}^{k}\sum_{m=0}^{j-k}\sum_{n=0}^{m}\left[3(k-i+1)(j-k-m+1)\cdot\hat{H}_{j-k-m+1}P_iP_{k-i+1}+(i+1)(k-i+1)\hat{H}_{j-k-m}P_{i+1}P_{k-i+1}\right]\cdot\hat{H}_n\hat{H}_{m-n}-A_kP_{j-k}\right\}+\sum_{k=0}^{j-1}\left[\sum_{i=0}^{k}\sum_{m=0}^{j-k}\sum_{n=0}^{m}(k-i+1)(k-i+2)\cdot\hat{H}_{j-k-m}\hat{H}_n\hat{H}_{m-n}P_iP_{k-i+2}+(j-k)(j-k+1)\hat{H}_0^3P_{k+1}P_{j-k+1}\right] \tag{27}$$

Unknown coefficients of the pressure expression (23) can be determined by the boundary conditions

$$P_0=P_i(p=p_i) \quad \text{at } Z=0(z=-L) \tag{28a}$$
$$\sum_{j=0}^{\infty}P_j=P_o(p=p_o) \quad \text{at } Z=1(z=L) \tag{28b}$$

where P_i is a dimensionless internal pressure and P_o a dimensionless external pressure. The first coefficient, P_0 of Eq. (23) is given by the equation (28a) and P_1 may be determined from Eq. (28) in the following manner: Substitute Eq. (26) for $j=0, 1, 2, \cdots$ into Eq. (28b) and truncate the infinite series at some N that gives acceptible accuracy. The truncated series is a polynomial in P_1 with one real, positive root that satisfies Eq. (28b). Once P_1 is known, the higher order coefficients such as P_2, P_3, $\cdots$ can be generated using the equations (26) and (27). Here

$$\hat{H}_0=1+\varepsilon_0\cos\theta+\varepsilon\cos(\theta-\Psi)+\tilde{h}/\bar{h} \tag{29a}$$
$$\hat{H}_1=-2\varepsilon\cos(\theta-\Psi) \tag{29b}$$
$$\hat{H}_2=0,\quad \hat{H}_3=0,\quad\cdots \tag{29c}$$

$$A_0=\frac{\xi_u n_u}{\Lambda_u\Gamma_1}\cos(n_u\theta-\Omega_u t+\phi)+\frac{\Gamma_2}{\Gamma_1}\left[\varepsilon_0\sin\theta+\varepsilon\sin(\theta-\Psi)-\frac{1}{\bar{h}}\frac{d\tilde{h}}{d\theta}\right] \tag{30a}$$

$$A_1=-\frac{2\varepsilon\Gamma_2}{\Gamma_1}\sin(\theta-\Psi) \tag{30b}$$

$$A_2=0,\quad A_3=0,\quad\cdots \tag{30c}$$

3.4 Leakage Flow Rate

The mass leakage rate in the axial direction is given by

$$\dot{m}=n\int_0^{\frac{2\pi}{n}}\int_{r_l}^{r_u}\rho r v_z dr d\theta \tag{31}$$

The dimensionless form of the above equation can be obtained by substituting the perfect gas law, the axial velocity in Eq. (18), the dimensionless parameters of Eq. (20), and pressure expression (23) into Eq. (31).

$$\dot{M}=\Gamma_1 n\int_0^{\frac{2\pi}{n}}\left[\hat{H}_0^3P_0P_1+(3\hat{H}_1P_0P_1+2\hat{H}_0P_0P_2+\hat{H}_0P_1^2)\hat{H}_0^2Z+\cdots\right]d\theta \tag{32}$$

where

$$\dot{M}=\frac{R_gT_rr_l\bar{h}}{8\eta_rU^2L^3}\dot{m} \tag{33}$$

Since the mass flow rate (32) cannot vary with Z it may be simplified with $Z=0$. According to Eq. (32), the leakage rate varies with the third power of the sealing gap and the second power of pressure. It indicates that the pressure distribution will also be an influential element to estimate the leakage rate for the compressible fluid flow. Eq. (32) can thus be simplified using the integration parameters, I_j.

$$\dot{M}=\Gamma_1P_0P_1(I_a+I_b+I_c+I_d+I_e) \tag{34}$$

where

$$I_a=2\pi$$

$$I_b = -3\xi_u \xi_l (\xi_u J_{11} + \xi_l J_{12})$$
$$I_c = 3|\hat{\varepsilon}|^2 \left[\pi - (\xi_u J_{31} + \xi_l J_{32}) \right]$$
$$I_d = 3\left[\pi(\xi_u^2 + \xi_l^2) + 2\,\xi_u \xi_l J_{13} + |\hat{\varepsilon}|(\xi_u^2 J_{36} + \xi_l^2 J_{37} + 2\xi_u \xi_l J_{38}) \right]$$
$$I_e = -6|\hat{\varepsilon}|(\xi_u J_{9u} + \xi_l J_{9l})$$

and $|\hat{\varepsilon}| = (\varepsilon_0^2 + \varepsilon^2 + 2\varepsilon_0 \varepsilon \cos \Psi)$

$$J_{11} = \delta(2n_u)(n_l)\frac{\pi}{2}\sin\left[\left(\Omega_l - 2\Omega_u\right)t + \phi\right]$$
$$J_{12} = \delta(2n_l)(n_u)\frac{\pi}{2}\sin\left[\left(\Omega_u - 2\Omega_l\right)t + \phi\right]$$
$$J_{13} = \delta(n_u)(n_l)\pi\cos\left[\left(\Omega_u - \Omega_l\right)t\right]$$
$$J_{36} = -\delta(2n_u)(1)\frac{\pi}{2}\cos\left(2\Omega_u t - \Psi - 2\phi\right)$$
$$J_{37} = -\delta(2n_l)(1)\frac{\pi}{2}\cos\left(2\Omega_l t - \Psi - 2\phi\right)$$
$$J_{38} = \frac{\pi}{2}\left\{\delta(n_l)(n_u+1)\cos\left[\left(\Omega_u - \Omega_l\right)t + \Psi\right] + \delta(n_u)(n_l+1)\cos\left[\left(\Omega_u - \Omega_l\right)t - \Psi\right]\right\}$$
$$J_{9l} = \delta(n_l)(1)\ \pi\sin\left(-\Omega_l t + \phi + \Psi\right)$$
$$J_{9u} = \delta(n_u)(1)\ \pi\sin\left(-\Omega_u t + \phi + \Psi\right)$$

where the symbol, δ_{ij} is defined as

$$\delta_{(i)(j)} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

4. RESULTS AND DISCUSSION

A numerical example will be presented to demonstrate the validity of the model and solution method developed. A typical shaft seal having the geometrical parameters and operating conditions was selected to demonstrate the validity of the model and solution method developed.

Shaft radius, r_2	6.35cm
Waviness magnitude, $\lvert\hat{h}\rvert$	4.1μm
Mean sealing gap, $\bar{h}$	40μm
Number of waves, n	2
Tilt parameter, ε	0.3
Shaft speed, N	50000rpm
Temperature difference, ΔT	500°C
Pressure difference, ΔP	$5.5\times10^5 N/m^2$

The material of shaft seals is shown for shaft (steel) -seal(carbon-graphite). The material properties are given in Table 1.

All the results presented in this paper were obtained for $\bar{h} = 40\mu$m, $U = 333$m/s and $\varepsilon_0 = 0.38$ unless otherwise stated.

Figure 3 shows a linear pressure drop in the axial direction for various values of L/r_2 ratios. The results of the pressure distribution along the axial direction are obtained from the recursive formula (26). Bryant and Kim(1987) showed a similiar pressure drop along the radial direction in the face seal.

Table 1 Material properties of the shaft and seal

	Shaft	Seal
Thermal expansion, mm/mm-°C	10.1×10^{-6}	5.2×10^{-6}
Thermal conductivity, W/m-°C	28.7	16

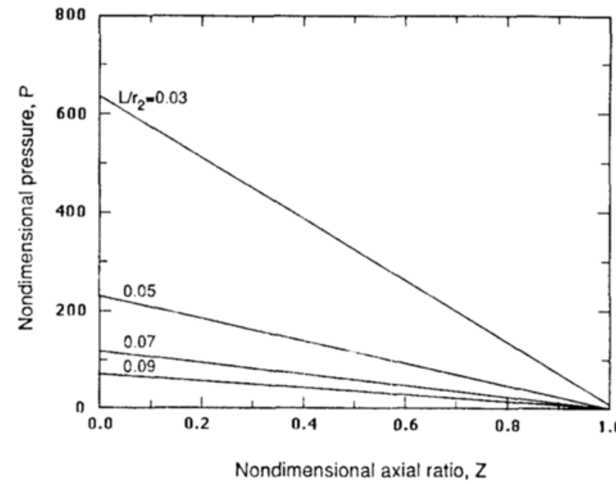

Fig. 3 Pressure distribution along the axial direction

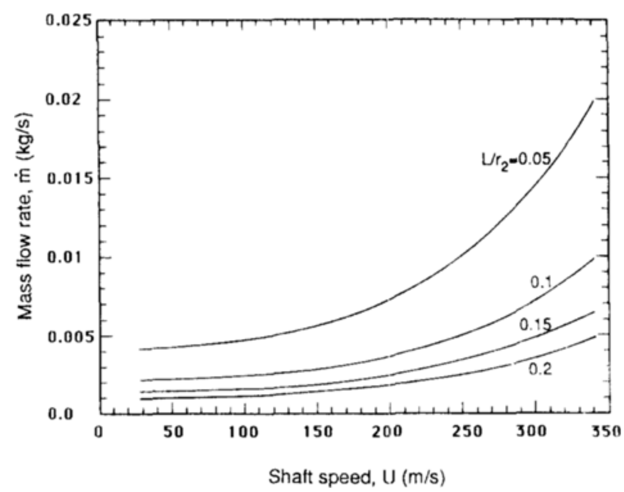

Fig. 4 Mass flow rate as a function of speed for various L/r_2 ratios

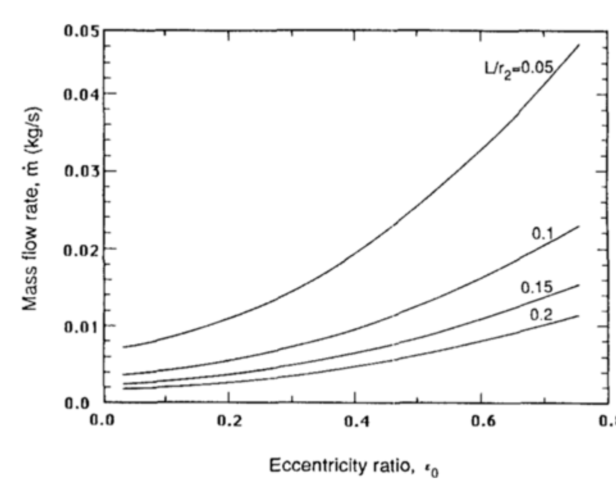

Fig. 5 Mass flow rate versus eccentricity ratio

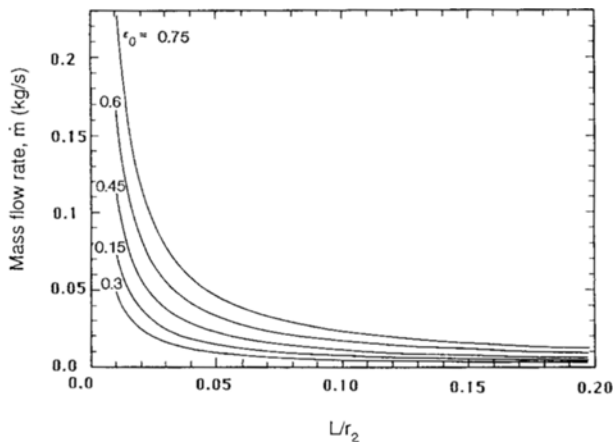

Fig. 6 Mass flow rate as a function of L/r_2 ratios for various values of eccentricity

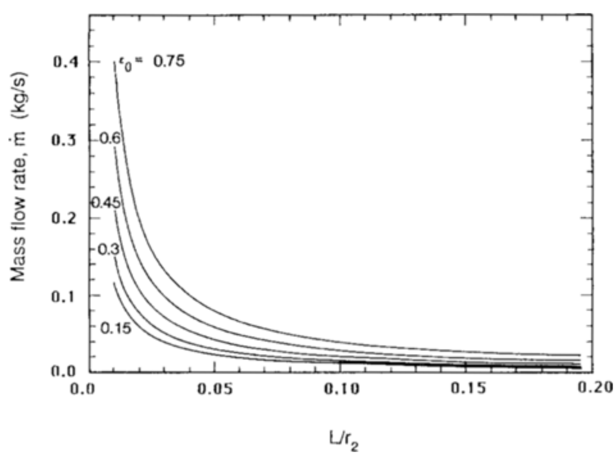

Fig. 7 Mass flow rate as a function of L/r_2 ratios for various values of eccentricity ; $U=333\text{m/s}$, $\bar{h}=7\times10^{-5}$

Figure 4 shows a substantial increase in leakage flow rate with increasing shaft speeds for the small width of the seal. When the shaft speed is around 150m/s the figure indicates the initiation of a sharp increase of the leakage flow rate. The mass flow rate has been plotted in Fig. 5 against the eccentricity ratio for a number of L/r_2. This figure shows the increase of leakage flow rate for the increased eccentricity. As the width of the seal increases, the mass flow rate shows a linearization.

In Figs. 6 and 7, the mass flow rate hyperbolically decreases as the ratio of L/r_2 increases for $\bar{h}=40\mu$m and 70 μm. As shown in Fig. 7, the high values of mean sealing gap will produce a remarkable increase in leakage rate. By examining the figures (4-7), it will be necessary to restrict the eccentricity and seal width to get an appreciable reduction of leakage flow rate.

5. CONCLUSIONS

An analytical method to estimate the leakage rate has been presented for compressible fluid flow across shaft seals with various sealing components such as the eccentricity, misalignment, and wavy surfaces.

Nonlinear Reynolds equation was solved using a power series method for the shaft seals. Sufficient accuracy is obtained considering the first four or five terms of the pressure distribution in a polynomial form.

According to the calculated results, the main factors which affects the seal performance at high speeds seem to be the width of the seal and eccentricity of the shaft. The result appears to show that the leakage flow rate is not a strong function of the surface waviness. Thus it is necessary to restrict the eccentricity ratio and lengthen the width of the seal for the acceptable accuracy of the seal performance. This can reduce the leakage flow rate even though the shaft speed is high.

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